MIXING IN VESSELS OF SQUARE CROSS-SECTION

V. Novák and F. Rieger

Faculty of Mechanical Engineering,
Czech Technical University, Prague 6

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The dependence was examined experimentally of the power input of a turbine and a three-paddle impeller at mixing of a low-viscosity liquid in a vessel of square cross-section. The effect was investigated of the relative size of the impeller and of the height of the impeller over the bottom on the power input and the homogenation effects. The results obtained were compared with the values for a cylindrical vessel with four baffles and it was found that the specific energy consumption for homogenation in a square vessel was lower than when using the same impeller in a cylindrical vessel with four baffles.

A majority of papers dealing with mixing of liquid media have been concerned with cylindrical vessels. Besides this type of mixing vessels, vessels of square cross-section have been sometimes used in industrial practice. Their advantage rests in a better utilization of space in production plants. The shape of the vessel suppresses to a considerable extent formation of the central vortex, and, accordingly, at current intensities of mixing no baffles or any other built-in facilities need be used. In contrast their disadvantage is that the square shape of the vessel is not suitable for pressure-vessels.

So far almost no experimental data have been available for the calculation of the power input and the homogenation effects of mixing in vessels of square cross-section. Of recently published papers only the paper by Bates, Fondy and Corpstein[1] mentions mixing in vessels of this type. The authors report that a central arrangement of the turbine impeller in a square vessel requires a 25% lower power input than the standard arrangement in cylindrical vessel with baffles.

EXPERIMENTAL

The aim of this study was to measure the power input and the homogenation effects of a flat blade disc turbine impeller (Fig. 1*a*) and a pitched blade turbine (Fig. 1*b*). A central arrangement of the impeller in a vessel of square cross-section without baffles was chosen, see Fig. 2. The ratio $H_2/A = 0{\cdot}3$ and $A/d = 3{\cdot}33$. The height of the liquid level was constant in all experiments and equal $H = A$. The effect was investigated of the A/d and H_2/A ratios on the power input and the homogenation effects. A fluid drive of the impeller enabled a continuous control of revolution from 15 to 1 600 r.p.m. A photoelectric pick-up Tesla BP 3 620 tachometer and a Tesla BM 362 counter were used for measuring the r.p.m. The measurements were carried out with both mentioned types of impellers of 60, 90 and 100 mm diameter in vessels of $A = 200$, 300 and 500 mm. Water, monoethylenglycol and aqueous solution of corn sirup were used in measurements.

The torque during mixing in vessels up to $A = 300$ mm in size was measured by placing the vessels on the turntable of a dynamometer shown in Fig. 3. To lower the passive resistances, the weight of the vessel and the liquid was compensated by pressure air brought into the space

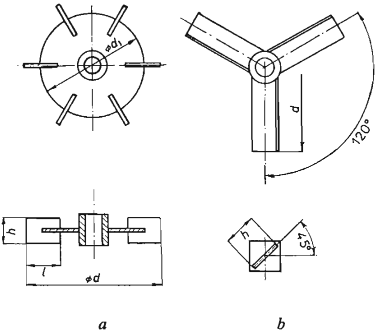

FIG. 1

Impellers Used

a Turbine impeller $h/d = 0{\cdot}2$, $l/d = 0{\cdot}25$, $d_1/d = 0{\cdot}75$, *b* pitched blade turbine $h/d = 0{\cdot}2$.

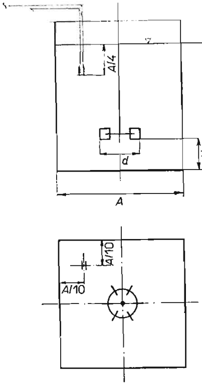

FIG. 2

Vessel and Impeller

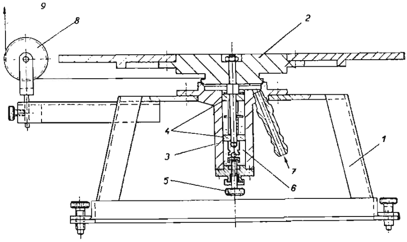

FIG. 3

Device for Measuring the Torque

1 Stand, *2* turntable, *3* casing, *4* ball bearings, *5* screw for adjusting the axial position of the plate, *6* lubricant, *7* pressure air inlet, *8* pulley with ball bearing, *9* string for transmission of detected force.

below the plate of the turntable. The shaft of the table was mounted by means of ball bearings filled with lubricant oil. The force acting on the circumference of the plate was transmitted by a silon string and a pulley to a balance. The circumference force could be detected on three different radii enabling the change of the range of the instrument. Using a table balance with the range 0—500 p, forces starting from 10 p could be measured. The corresponding minimum torque is $2{\cdot}45 \,.\, 10^{-3}$ Nm, if the force is acting on the radius $r = 25$ mm. The maximum torque on a balance with weightability 5000 p was 4·78 Nm for the maximum radius $r = 97{\cdot}5$ mm. To measure the power input in the vessel of the maximum diameter $A = 500$ mm, a torque dynamometer Sfindex, Switzerland, was used with two ranges: 0—12 Nm and 0—40 Nm. The torque was measured on the shaft of the impeller driven by an electric motor with the r.p.m. controllable in the range 250—1 600. To measure the homogenation, a conductivity method described earlier in the literature[2,3] was used. The conductivity probe consists of two platinum wires 0·8 mm diameter shaped into a square 8 × 10 mm. The distance of the electrode was 8 mm. The location of the probe in the vessel is sketched in Fig. 2. The signal from the probe was brought into a detector of small variations of conductivity and its time course after adding a tracer was recorded on an EKNT 1/D chart recorder. The width of the chart was 250 mm, the time of traverse less than 1 s and the maximum speed of the chart was 36000 mm/h. The added tracer was a 4% aqueous solution of NaCl injected below the level of liquid near the shaft of the impeller. The added volume of the tracer was varied between 0·5 and 4 cm^3. As the time of homogenation, Θ, was taken the time elapsed before all conductivity fluctuations were damped below ±2% of the total difference of conductivity. Each measurement of the homogenation time was repeated 10 times and the arithmetic average was taken for the result. The experimental set-up has been described earlier[4]. The number of measurements of the power input was over 400 and the total number of measurements of the time of homogenation was greater than 1 400. The results were processed statistically.

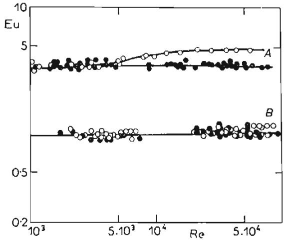

FIG. 4

Power Input Characteristics of Impellers Used

○ Cylindrical vessel with four radial baffles, $D/d = 3{\cdot}33$, $H_2/D = 0{\cdot}3$; ● square vessel without baffles $A/d = 3{\cdot}33$, $H_2/A = 0{\cdot}3$; *A* turbine impeller, *B* pitched blade impeller.

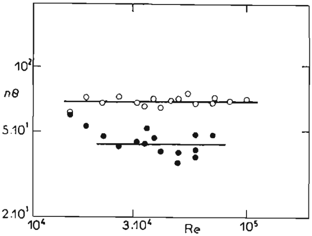

FIG. 5

Dependence $n\Theta = f(\mathrm{Re})$, Vessel of Square Cross-Section

$A/d = 3{\cdot}33$, $H_2/A = 0{\cdot}3$, ● turbine impeller, ○ pitched blade impeller.

RESULTS AND DISCUSSION

The results of measurement of the power input were processed to give the usual dependence of the Euler power number on the Reynolds number

$$(P/\varrho n^3 d^5) = f_1(nd^2\varrho/\mu)\,. \tag{1}$$

A majority of measurements were carried out in the turbulent region, where the Euler number is independent of the Reynolds group and one can write

$$P/\varrho n^3 d^5 = k_1\,, \tag{2}$$

where the constant k_1 depends on the geometry of the system only.

The results of measurements of the time of homogenation were also processed into the usual dimensionless relation of the product $n\Theta$ and the Reynolds number

$$n\Theta = f_2(nd^2\varrho/\mu)\,. \tag{3}$$

In the turbulent region the dependence of $n\Theta$ on the Reynolds number is again insignificant and we have

$$n\Theta = k_2\,, \tag{4}$$

where the magnitude of the constant k_2 depends again solely on the geometry parameters of the system. A following dimensionless relation was used for comparison of the investigated types of arrangements and impellers from the viewpoint of energy consumption:

$$(E\Theta/\mu D_e^3) = f_3(D_e^2\varrho/\mu\Theta)\,. \tag{5}$$

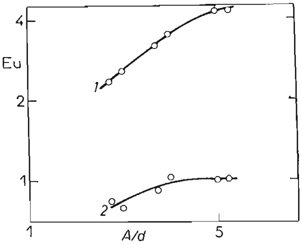

FIG. 6

Dependence Eu $= f(A/d)$, Vessel of Square Cross-Section

$H_2/A = 0{\cdot}3$, *1* turbine impeller, *2* pitched blade impeller.

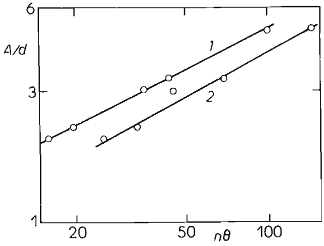

FIG. 7

Dependence $n\Theta = f(A/d)$

$H_2/A = 0{\cdot}3$, *1* turbine impeller, *2* pitched blade impeller.

The magnitude of the energy criterion on the left hand side of Eq. (*5*) is a measure of the energy of homogenation related to the volume capacity of the equipment. In order that a comparison with a cylindrical vessel of standard arrangement $H/D = 1$ (which for $A = D$ has a smaller volume that the square vessel) can be made, the equivalent diameter, D_e, must be substituted into the relations for vessels of non-circular cross-sections. The equivalent diameter is defined as

$$D_e = A(\pi/4)^{1/3} . \quad (6)$$

The most suitable for homogenation will thus be an impeller which at a requested volume capacity of the mixing equipment will homogenize the liquid at minimum consumption of energy. This means an impeller characterized at a given value of $D_e^2\varrho/\mu\Theta$ by a minimum value of the criterion $E\Theta/\mu D_e^3$.

Fig. 4 plots the results of comparative measurements of the power input as a dependence Eu = f(Re) in vessels of circular and square cross-section for basic arrangements and both types of impellers used in the region Re $> 10^3$. From the characteristic of the turbine impeller (curve *A*) it is evident that in the turbulent region of mixing the power input for a square vessel is lower than that for a cylindrical vessel and standard arrangement with four radial baffles. The difference amounted in our case to 26·6% which is in good agreement with the value 25% reported by Bates and coworkers[1]. The curve *B* in the same figure represents the values measured with the pitched blade turbine. The values of the Euler criterion measured in the square vessel are somewhat smaller than those for the cylindrical vessel but the differences are statistically insignificant.

The average values of the Euler number and 95% confidence limits for mixing in square vessels obtained by processing the experimental data (basic arrangement $A/d = 3{\cdot}33$, $H_2/A = 0{\cdot}3$) from the region Re $> 10^4$ are for the turbine impeller:

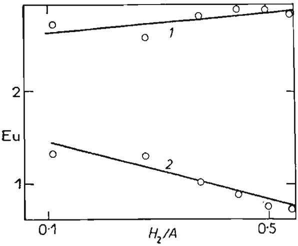

FIG. 8

Dependence Eu $= f(H_2/A)$

$A/d = 3{\cdot}33$, *1* turbine impeller, *2* pitched blade impeller.

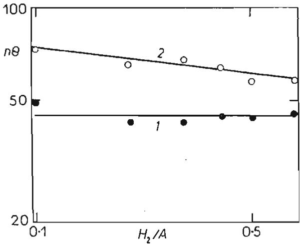

FIG. 9

Dependence $n\Theta = f(H_2/A)$

$A/d = 3{\cdot}33$, turbine *1* impeller, *2* pitched blade impeller.

$$(P/\varrho n^3 d^5) = 3{\cdot}51 \pm 0{\cdot}064, \quad S = 0{\cdot}1465, \quad m = 22, \qquad (7)$$

and for the pitched blade turbine.

$$(P/\varrho n^3 d^5) = 1{\cdot}024 \pm 0{\cdot}031\,, \quad S = 0{\cdot}071\,, \quad m = 23\,. \qquad (8)$$

Fig. 5 plots the results of measurement of the homogenation time as a dependence $n\Theta = f(\mathrm{Re})$ for basic arrangement a square vessel and both types of impellers. In the region of developed turbulence the dependence on the Reynolds number is statistically insignificant. The experimental values were again processed to give the average values and 95% confidence limits of $n\Theta$ at basic arrangement $A/d = 3{\cdot}33$ and $H_2/A = 0{\cdot}3$. Thus for the turbine impeller in the region $\mathrm{Re} > 2{\cdot}10^4$:

$$n\Theta = 43{\cdot}1 \pm 2{\cdot}51\,, \quad S = 4{\cdot}34\,, \quad m = 14\,, \qquad (9)$$

and for the pitched blade turbine and $\mathrm{Re} > 10^4$:

$$n\Theta = 68{\cdot}4 \pm 1{\cdot}84\,, \quad S = 3{\cdot}47\,, \quad m = 16\,. \qquad (10)$$

Each plotted point represents an average of ten measurements. In addition, the effect was examined of the relative size of the impeller and the relative height of the impeller over the bottom on the power input and the homogenation time. All measurements of the power input and the homogenation time in the course of investigation of the effect of the geometry were carried out in the turbulent region where Eu and $n\Theta$ were found in all cases to be independent of the Reynolds group. The effect of A/d was examined at constant ratio $H_2/A = 0{\cdot}3$. A plot of the $\mathrm{Eu} = f(A/d)$ dependence for the turbine and the pitched blade impeller is shown in Fig. 6.

The next effect studied was that of the A/d ratio on the homogenation time at constant $H_2/A = 0{\cdot}3$. Fig. 7 shows the averages of $n\Theta$ for given experimental values of A/d. It was established that in the turbulent region and A/d ranging between 2 and 5 the last two quantities can be correlated by the following power expression

$$n\Theta = K_1(A/d)^\alpha\,. \qquad (11)$$

The value of K_1 and the exponent α were obtained by the least square technique. For the turbine impeller and $\mathrm{Re} > 2{\cdot}10^4$: $K_1 = 3{\cdot}82$, $\alpha = 2{\cdot}014$, $S_{\log K_1} = 0{\cdot}0358$, $S_\alpha = 0{\cdot}0663$, $m = 25$; for the pitched blade impeller and $\mathrm{Re} > 10^4$: $K_1 = 7{\cdot}71$, $\alpha = 1{\cdot}796$, $S_{\log K_1} = 0{\cdot}03529$, $S_\alpha = 0{\cdot}066$ $m = 39$.

The effect of the remaining parameter – the ratio H_2/A – on the power input is shown in Fig. 8, plotting the average values of Eu for individual experiment a 1 values of H_2/A and constant $A/d = 3{\cdot}33$. For both types of impellers and H_2/A

ranging between 0·1 and 0·6 this relation can be expressed by the power expression

$$\mathrm{Eu} = K_2(H_2/A)^\beta\,. \tag{12}$$

The values of the constant K_2 and the exponent β obtained by processing the experimental data are as follows: For the turbine impeller and $\mathrm{Re} > 10^4$: $K_2 = 3{\cdot}76$, $\beta = 0{\cdot}073$, $A/d = 3{\cdot}33$, $S_{\log K_2} = 0{\cdot}00768$, $S_\beta = 0{\cdot}0128$, $m = 100$; for the pitched blade impeller and $\mathrm{Re} > 10^4$: $K_2 = 0{\cdot}748$, $\beta = -0{\cdot}254$, $A/d = 3{\cdot}33$, $S_{\log K_2} = 0{\cdot}00871$, $S_\beta = 0{\cdot}015$, $m = 92$.

The power input of the turbine impeller increases slightly with increasing H_2/A ratio. Despite of the fact that the value of the exponent β is relatively low (0·073), it was found to be statistically significant at 5% significance level.

In contrast, the power input of the pitched blade turbine decreases with increasing H_2/A ratio. Both trends are in qualitative agreement with the mutual relation of Eu and H_2/D in a cylindrical vessel with similar types of impellers reported by Bates and coworkers[1]. Fig. 9 plots the experimental dependence $n\Theta = \mathrm{f}(H_2/A)$. The effect of H_2/A on $n\Theta$ can be again expressed by a power expression of the following type

$$n\Theta = K_3(H_2/A)^\gamma\,. \tag{13}$$

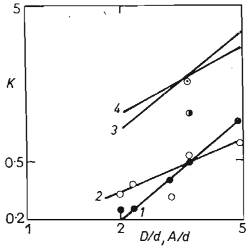

FIG. 10

Dependence of K on A/d or D/d Ratio

1 Turbine impeller, square vessel, *2* pitched blade impeller, square vessel, *3* pitched blade impeller, cylindrical vessel[5], *4* pitched blade impeller with 6 blades, cylindrical vessel[5], ⊙ turbine impeller, cylindrical vessel, ◑ pitched blade impeller, $H_2/A = 0{\cdot}3$.

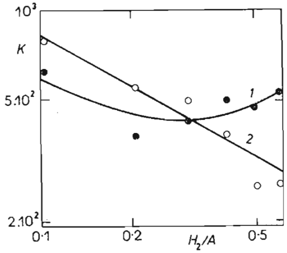

FIG. 11

Dependence of K on H_2/A

Square vessel, $A/d = 3{\cdot}33$, *1* turbine impeller, *2* pitched blade impeller.

At constant $A/d = 3{\cdot}33$ and $0{\cdot}1 \leqq H_2/A \leqq 0{\cdot}6$, the values found for the turbine impeller and $\mathrm{Re} > 2{\cdot}10^4$ are: $K_3 = 40{\cdot}1$, $\gamma = -0{\cdot}055$, $S_{\log K_3} = 0{\cdot}0192$, $S_\gamma = 0{\cdot}031$, $m = 32$. On a 5% significance level the value of the exponent γ is statistically insignificant and it may be assumed that $n\Theta$ for turbine impellers is independent of H_2/A. For the pitched blade turbine at $\mathrm{Re} < 10^4$ we get: $K_3 = 55{\cdot}12$, $\gamma = -0{\cdot}02$, $S_{\log K_3} = 0{\cdot}014$, $S_\gamma = 0{\cdot}022$, $m = 36$. A decrease of $n\Theta$ with increasing H_2/D for a pitched blade impeller in a cylindrical vessel with four baffles was found also by Kvasnička[5].

The results of measurement of the power input and the homogenation time were further utilized for comparison of both types of impellers and the geometric arrangement from the standpoint of energy necessary for homogenation. To calculate the energy criterion a following relation was used

$$\frac{E\Theta}{\mu D_e^3} = (n\Theta)^3 \frac{P}{\varrho n^3 d^5} \left(\frac{d}{D_e}\right)^5 \frac{D_e^2 \varrho}{\mu\Theta} . \tag{14}$$

Since all measurements were carried out in the turbulent region where Eu and $n\Theta$ were constant and independent of Re, one can write

$$(E\Theta/\mu D_e^3) = K(D_e^2\varrho/\mu\Theta) . \tag{15}$$

The constant K of this relation depends then only on the geometry simplexes. The functions $K = f(A/d)$ and $K = f(H_2/A)$, shown in Figs 10 and 11, permit following conclusions to be made:

A position closer to the bottom (H_2/A between 0·2 and 0·3) appears most suitable for homogenation with the turbine impeller, which corresponds also to the position $H_2 = d$ used most frequently for mixing in cylindrical vessels.

The upper position appears most suitable for the pitched blade impeller. In fact, from energy viewpoint, it is suitable to locate the impeller in the upper half of the vessel. On the other hand, a small distance from the level carries the danger of entraining air. Consequently, the recommendable position is up to $H_2/A = 0{\cdot}4-0{\cdot}5$.

As to the effect of the A/d ratio, as most economic for energy appear low values of A/d, which, on the other hand, are accompanied by increased costs of the impeller and the drive.

Fig. 10 gives a comparison of the results measured by Kvasnička[5] in a cylindrical vessel with a three-blade and a six-blade turbine with pitched blades. The values shown by circles are our results measured by the turbine and pitched blade impellers ($D/d = 3{\cdot}33$, $H_2/D = 0{\cdot}3$). It is seen that the energy necessary for homogenation of the same volume of liquid in a vessel of square cross-section is smaller by about a factor of three in comparison with a cylindrical vessel with four radial baffles. This difference is caused on one hand by a better utilisation of the space (at $A = D$

and the same height of liquid the square vessel has a greater volume), and by a lower power input (using the turbine impeller) and shorter time of homogenation on the other hand.

LIST OF SYMBOLS

A	size of square vessel (m)
d	diameter of impeller (m)
D	inner diameter of vessel (m)
D_e	equivalent diameter of square vessel (m)
$E = P\Theta$	energy necessary for homogenation (Ws)
Eu	modified Euler number for mixing (power number)
H_2	height of lower edge of impeller over bottom (m)
m	number of experiments
n	revolutions of impeller per unit time (s^{-1})
P	power input (W)
Re	modified Reynolds number for mixing
S	standard deviation
μ	dynamic viscosity ($N s m^{-2}$)
ϱ	density of liquid ($kg m^{-3}$)
Θ	time of homogenation (s)

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